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Verification of the Draper Semianalytic Satellite Theory in OREKIT

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Rev 5

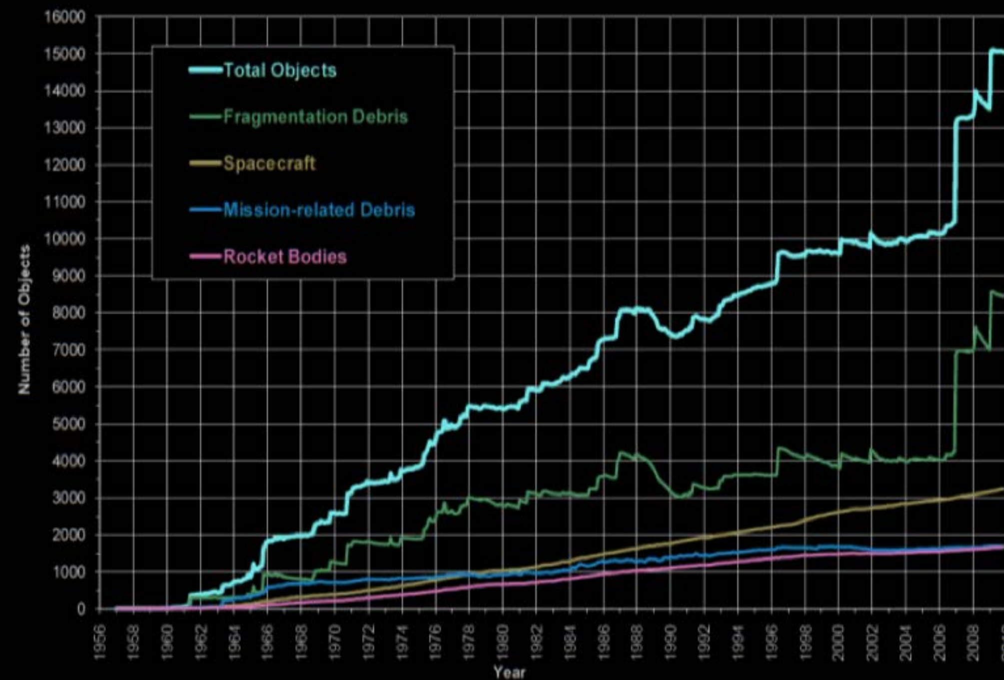
Outline

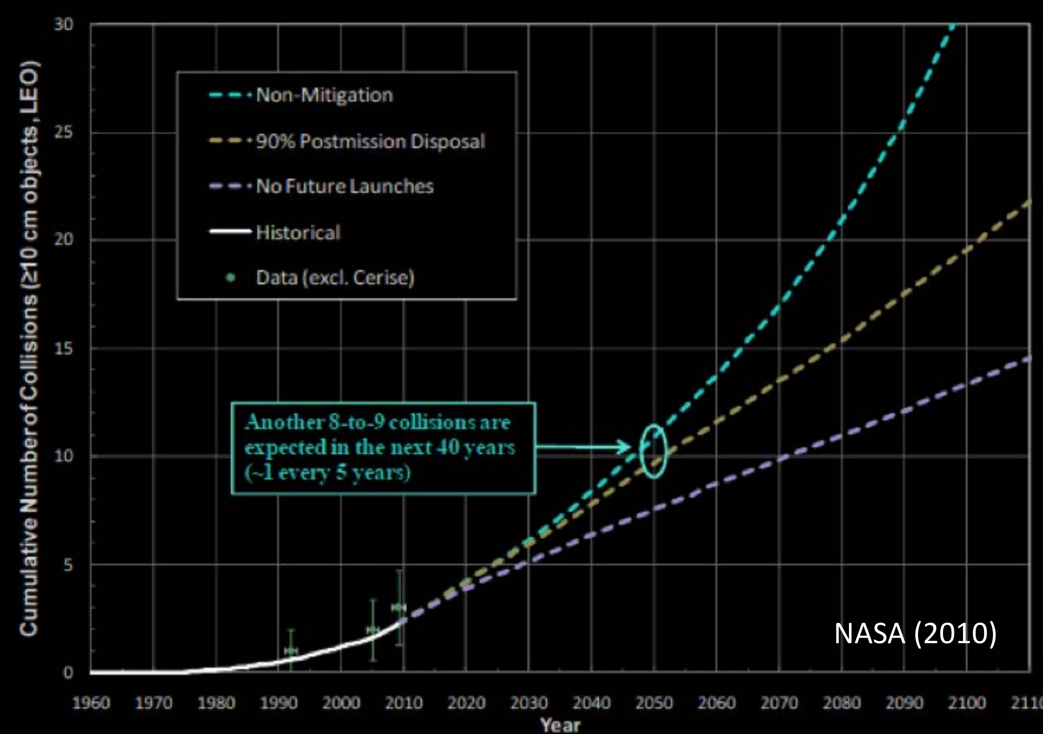
- Space Situational Awareness (SSA)
- Legacy astrodynamics software
- Orbit extrapolation kit (OREKIT)
- The Draper Semianalytic Satellite Theory (DSST)
- OREKIT implementation of DSST
- System testing

The Problem:

- Increasing space debris
- Estimated number of uncatalogued items: ~60 000

  Osiander & Ostdiek (2009)
- Legacy *(proprietary)* software systems





NASA (2010)

Astrodynamics software

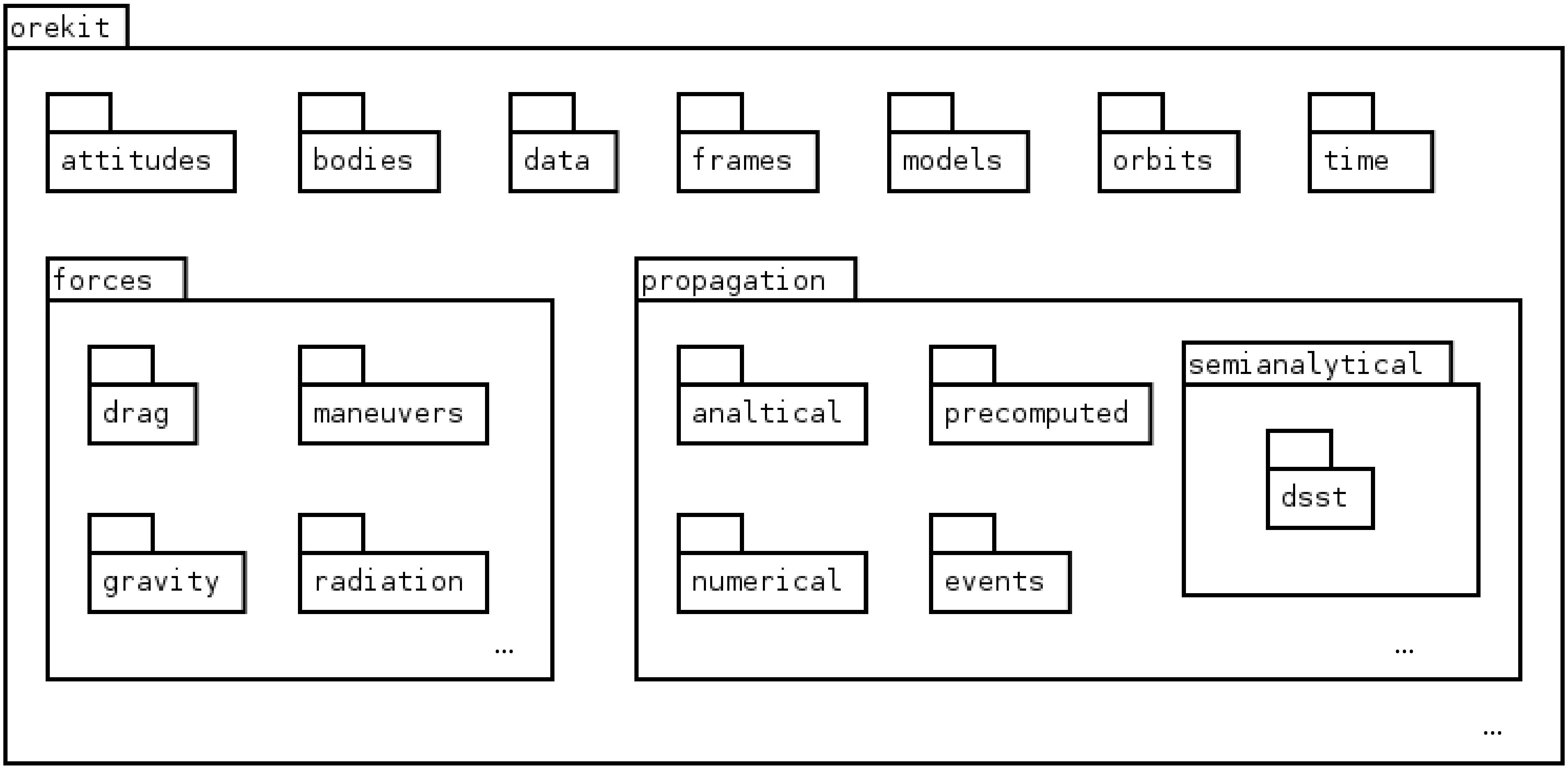
- Legacy software:
  - Increasing maintenance costs
  - Slow (or no) update lifecycle
  - Bound by out-dated hardware or design principles
- Proprietary software:
  - Less transparency
  - Difficult to audit or update
  - Less R&D
- Solution: Open Source astrodynamics software

OREKIT *(**OR**bit **E**xtrapolation **KIT**)*

- Open Source astrodynamics library in Java, developed at CS
- Tested on rendezvous between ATV and International Space Station
- Framework allows easy use of multiple:
  Time scales, Frames, Data formats, Propagators
- Growing developer base
- Can be modified for commercially use (Apache License)
- Free and *user extensible*

Functional decomposition in OREKIT

- **Object oriented:** focus on objects rather than processes
- **Encapsulation:** *plug-and-play* orbit construction



DSST migration

- Possible to migrate algorithms from legacy software to OREKIT framework
- Starting with Draper Semianalytic Satellite Theory (DSST)
- SST development started in 1970s at NASA GSFC. Continued at Draper Laboratory and recently MIT Lincoln
- Access to DSST is limited. Development of web interface, but source is closed and in Fortran 77

What is semianalytic satellite theory?

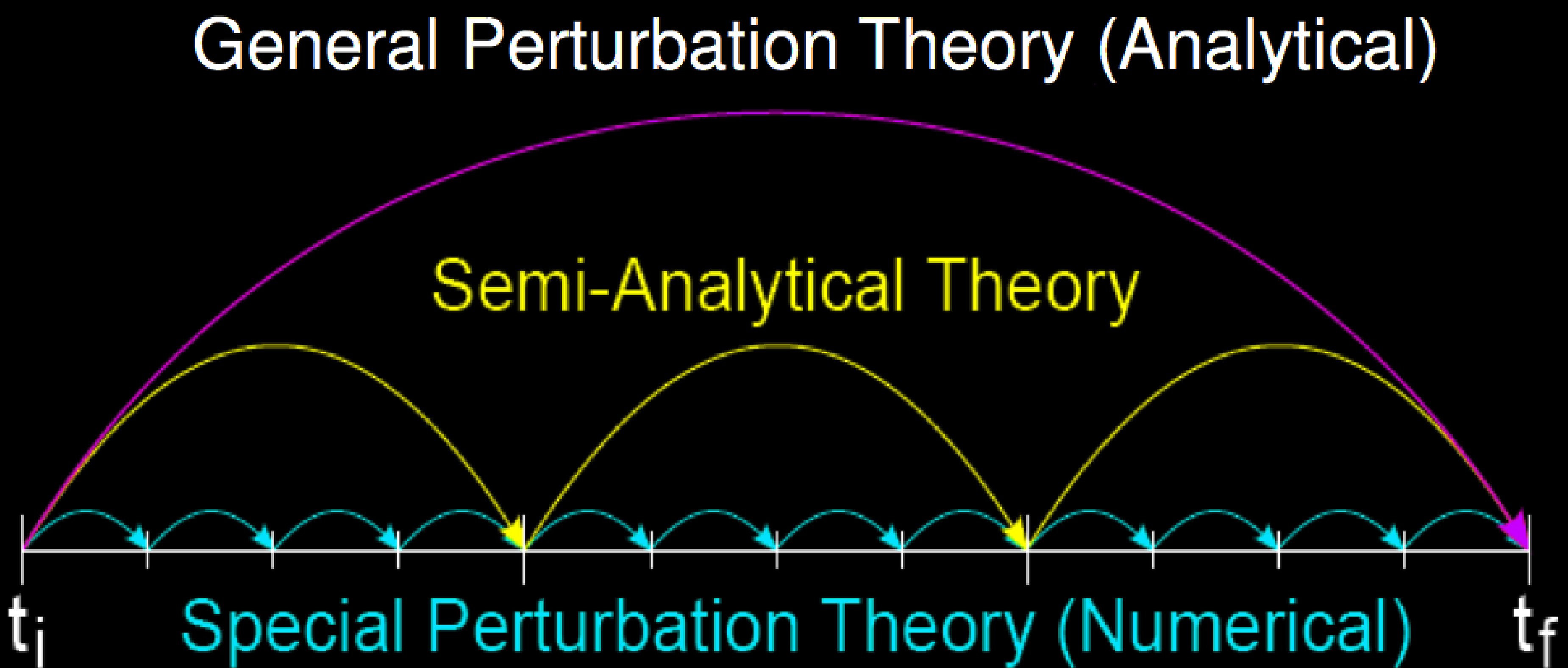
- Generalised Method of Averaging used to decouple long and short period motion
- Treat contributions separately and integrate long-period motion with larger step size
- Increases computational efficiency while retaining accuracy
- Useful for resource-intense tasks such as SSA

Draper Semi-analytical Satellite Theory (DSST)

Comprehensive force models:

- Zonal harmonics (thru J_{50})
- Tesseral harmonics (50x50 field)
- J_2-squared terms
- Lunar-solar point-masses
- Solid Earth tides
- Atmospheric drag
- Solar radiation pressure

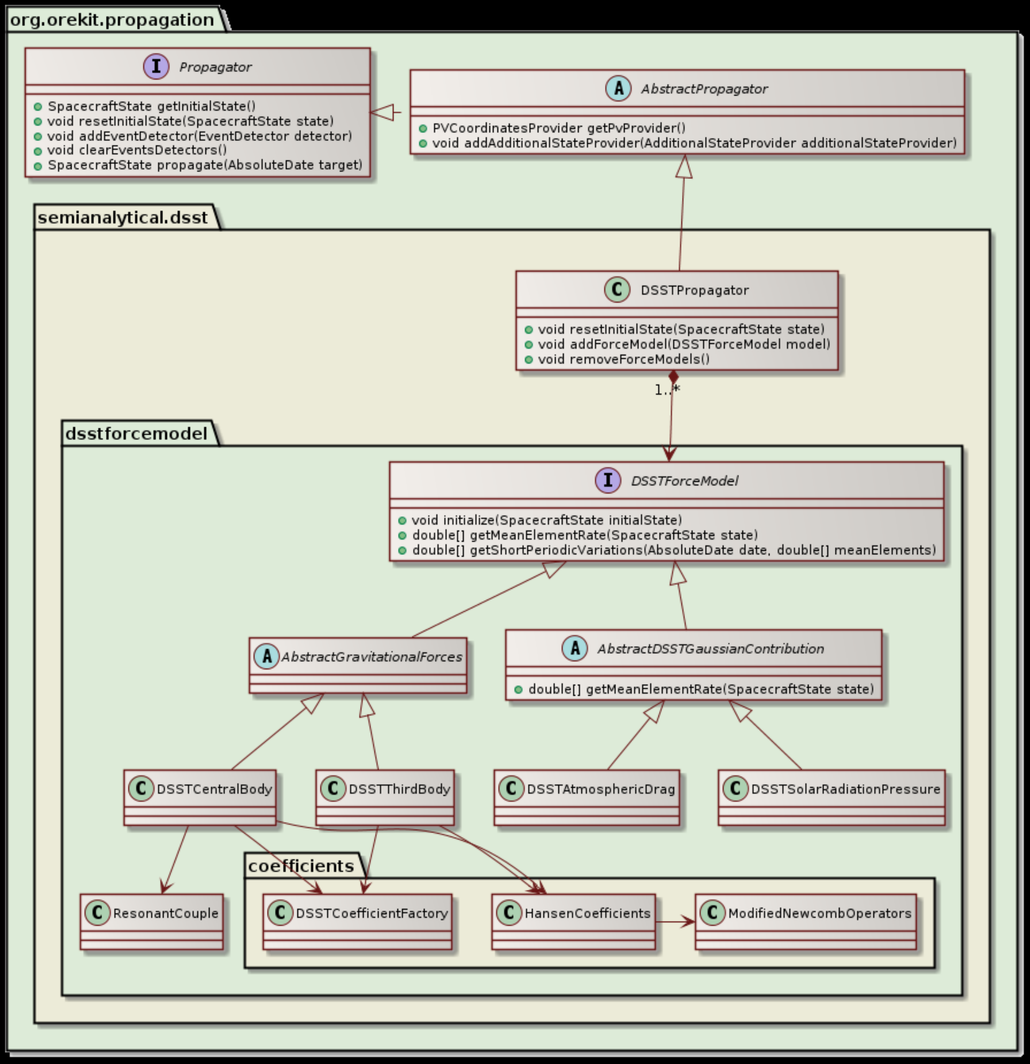
Comparison of integration grids



J. F. San Juan (2012)

OREKIT implementation

- Current implementation based on Danielson *et al.* (1995)
- Refactored to fit the OREKIT framework (Object Oriented Design)
- Interoperability with other OREKIT classes
- Undergoing verification



Semianalytic Satellite Theory Document–

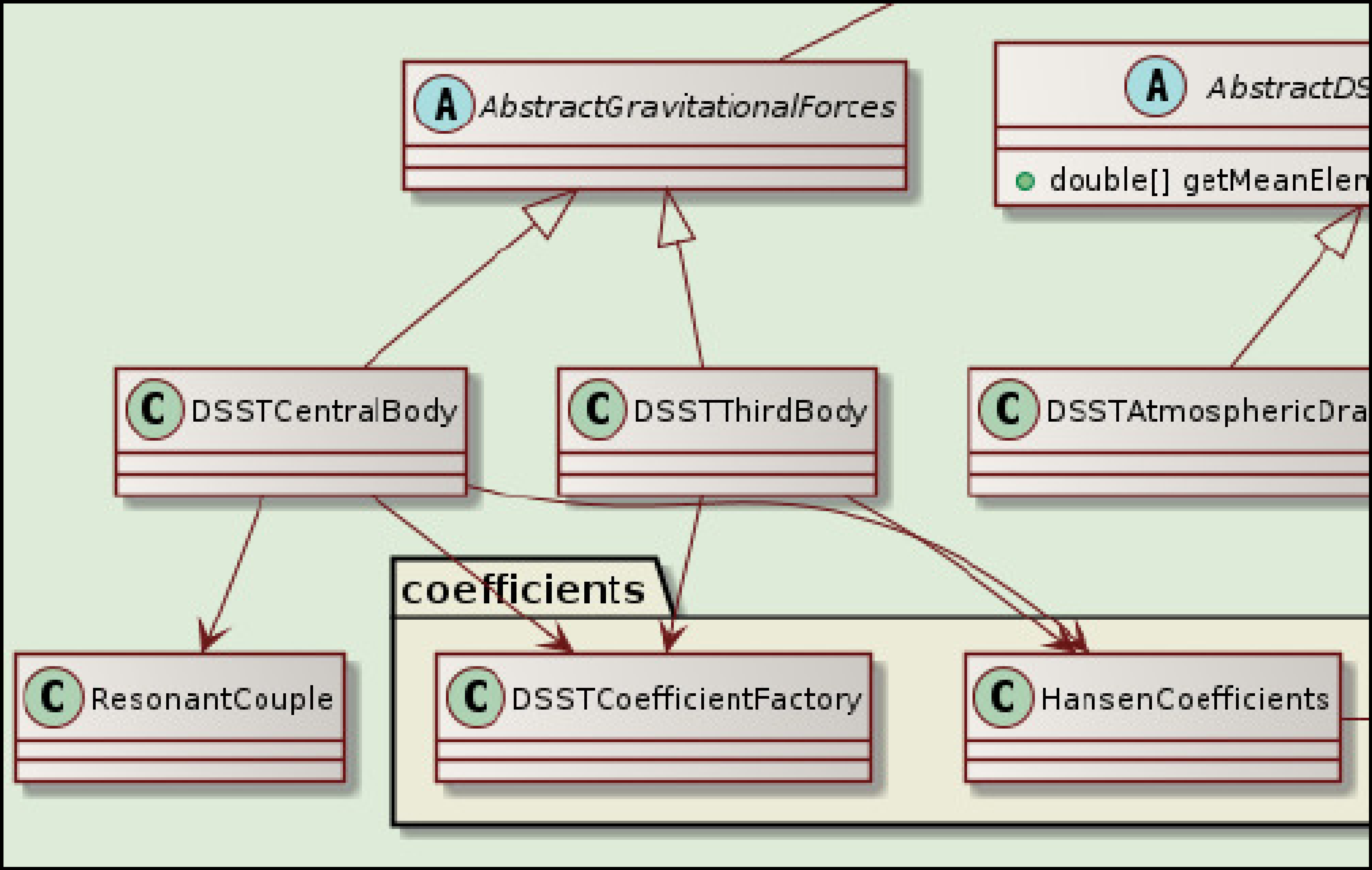
D. Danielsen, C. Sagovac, B. Neta, L. W. Early

- Naval Postgraduate School, Monterey, CA
- Dr. Steve Knowles of the Naval Space Command gave the initial impetus for the generation of this document
- The purpose was to simplify, assemble, unify, and extend the DSST theory whose documentation was only available in conference preprints, published papers, technical reports (including MIT theses), and private communications
- Report developed around 1993-95 (theory development from 1974).

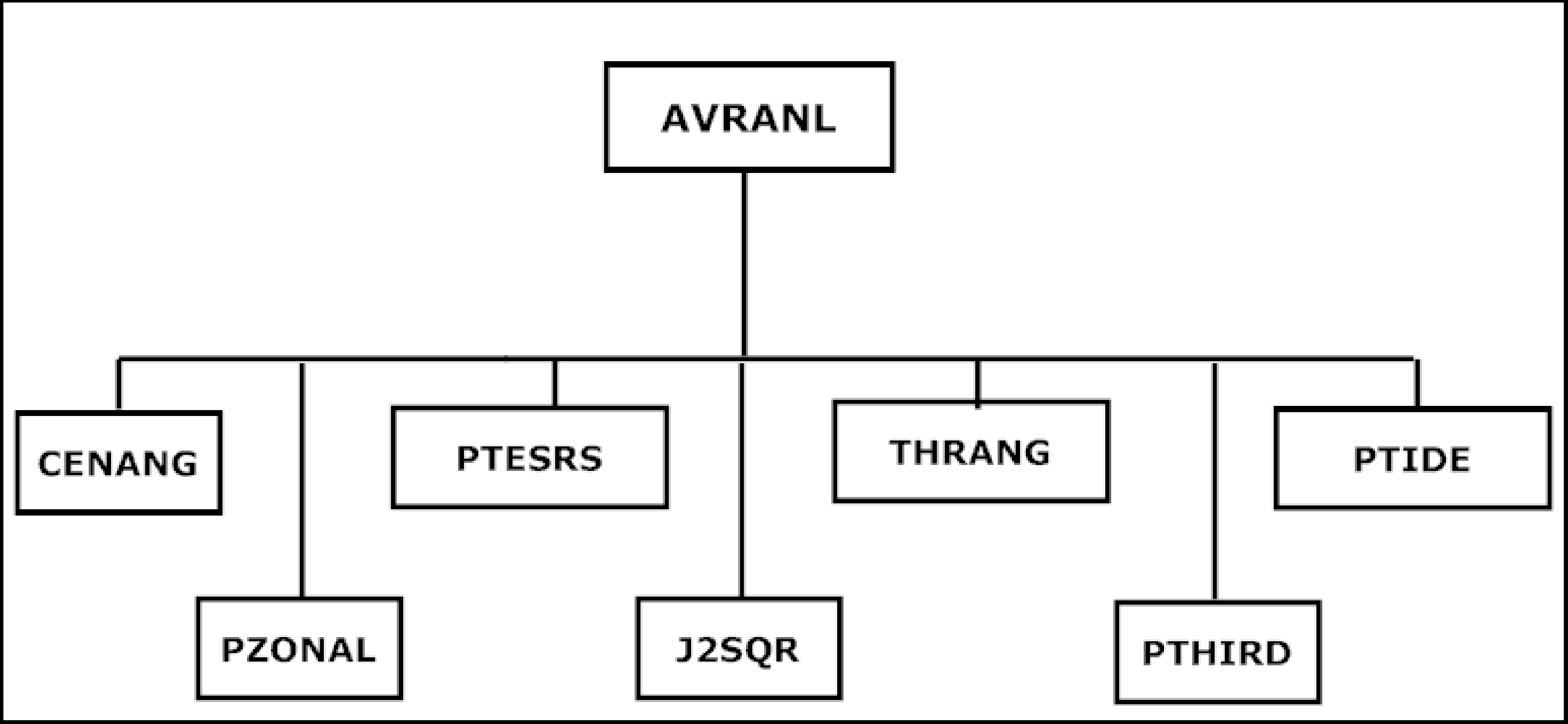
Semianalytic Satellite Theory Doc. Cont'd

- Outline
  - Mathematical Preliminaries
  - First-Order Mean Element Rates
  - First-Order Short-Periodic Variations
  - Higher-Order Terms
  - Truncation Algorithms
  - Numerical Methods
- The NPS report introduces modifications to the formulation at secondary levels
- The NPS report has not previously been used as the basis for software

Drilling Down into the Orekit dsst-propagator-class-diagram



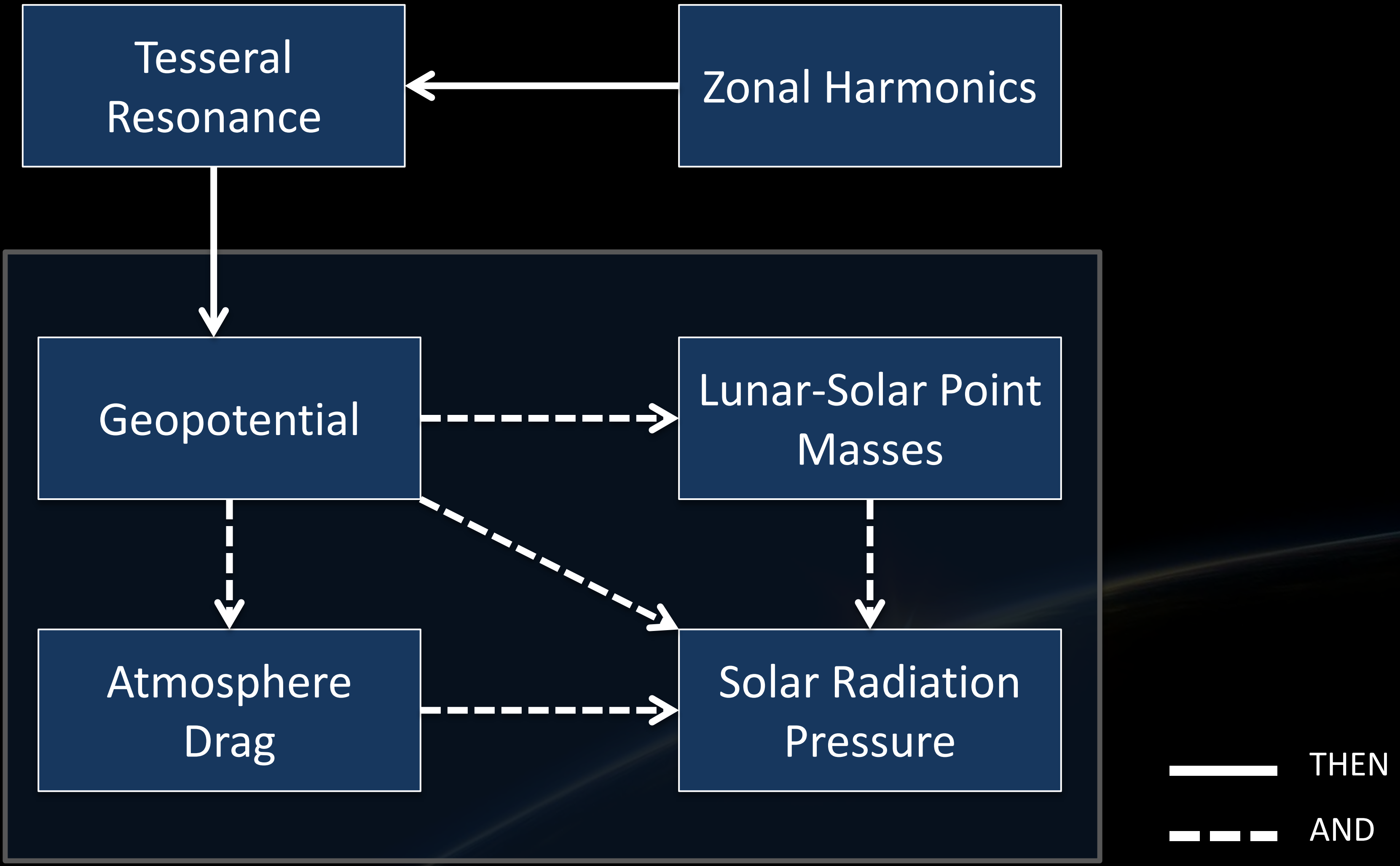
F77 DSST Standalone sequential decomposition of the mean element rates



Verification strategy

- Compare OREKIT against DSST F77 (DSST Standalone & GTDS DSST)
- Use interactive debuggers in Java and Fortran to locate any discrepancies
- Align physical models between systems
- Generate test cases for DSST functionality

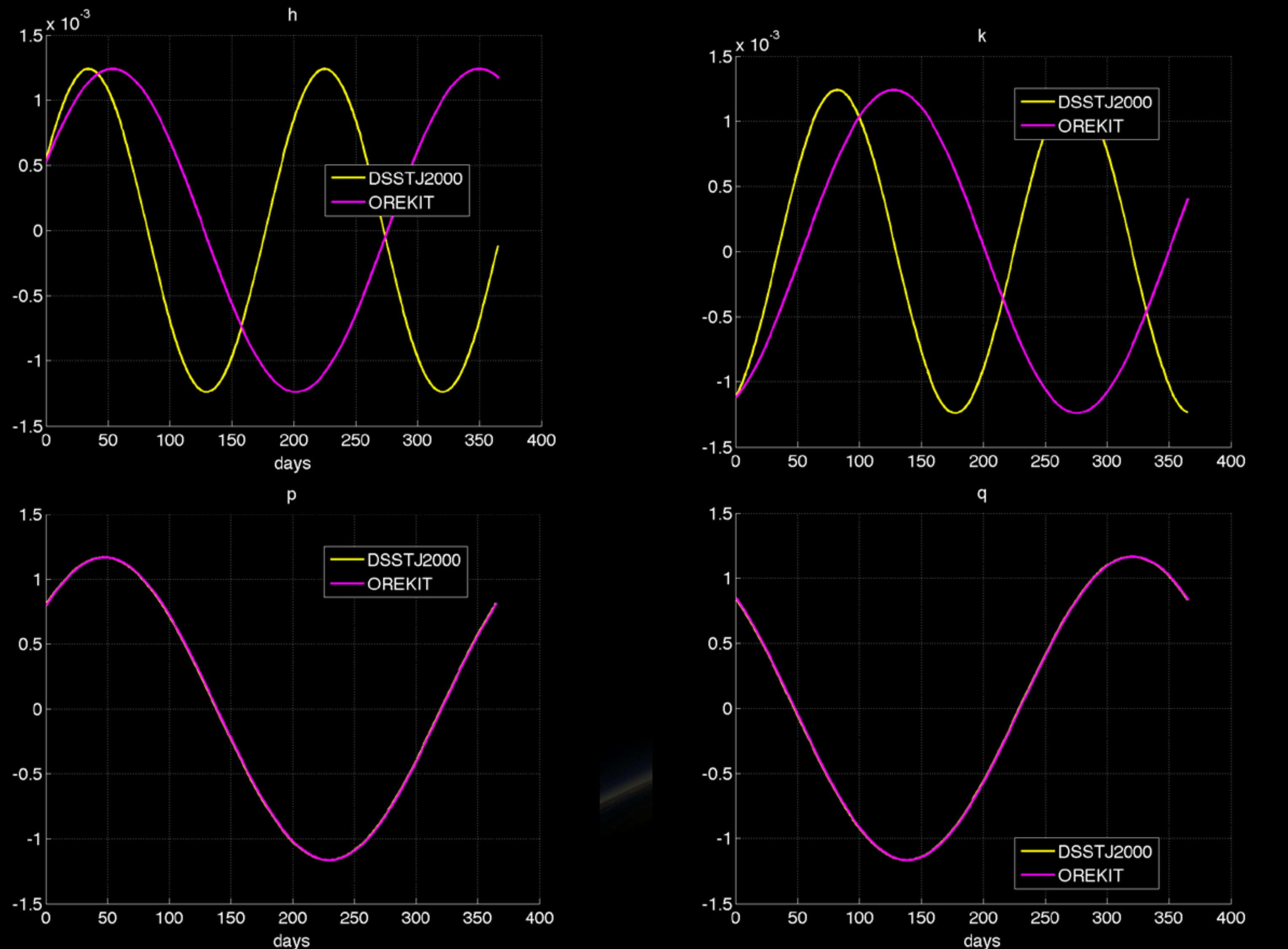
DSST test plan



Orekit DSST vs F77 DSST

- Input & output coordinate system: J2000
- Input parameters:
  - Mean Keplerian elements or
  - Mean Equinoctial elements (a, h, k, p, q, λ)
- Integration Coordinate System: J2000
- Integration Method: Runge-Kutta with Predictor-Corrector as backup
- Output: (1) mean Keplerian elements, (2) mean equinoctial elements, and (3) position and velocity

Test 1: LEO, 2 x 0
Equinoctial element histories over 365 days



Resolving the anomaly in the J2-only mean element case

- Comparison of debug output from Orekit routine DSSTCentralBody.java and from F77 DSST routines AVRANL/PZONAL
  - p dot and q dot approximately agree
  - h dot and k dot do not agree
- **Matlab** and **maxima** models for the equations of motion
- The partials of the potential wrt h and k are likely sources of the error

Danielsen Section 3.1, Eq (1): averaged eqs of motion for zonal harmonics

$$\frac{da}{dt} = 0$$

$$\frac{dh}{dt} = \frac{B}{A}\frac{\partial U}{\partial k} + \frac{k}{AB}(pU_{,\alpha\gamma} - IqU_{,\beta\gamma})$$

$$\frac{dk}{dt} = -\frac{B}{A}\frac{\partial U}{\partial h} - \frac{h}{AB}(pU_{,\alpha\gamma} - IqU_{,\beta\gamma})$$

$$\frac{dp}{dt} = -\frac{C}{2AB}U_{,\beta\gamma}$$

$$\frac{dq}{dt} = -\frac{IC}{2AB}U_{,\alpha\gamma}$$

$$\frac{d\lambda}{dt} = -\frac{2a}{A}\frac{\partial U}{\partial a} + \frac{B}{A(1+B)}(h\frac{\partial U}{\partial h} + k\frac{\partial U}{\partial k}) + \frac{1}{AB}(pU_{,\alpha\gamma} - IqU_{,\beta\gamma})$$

Partial derivatives of the potential due to zonals (Danielsen, Sect. 3, eq. 6)

$$
\begin{aligned}
\frac{\partial U}{\partial a} &= \frac{\mu}{a^2}\sum_{s,n}(2-\delta_{0s})(n+1)\left(\frac{R}{a}\right)^n J_n V_{ns} K_0^{-n-1,s} Q_{ns} G_s \\
\frac{\partial U}{\partial h} &= -\frac{\mu}{a}\sum_{s,n}(2-\delta_{0s})\left(\frac{R}{a}\right)^n J_n V_{ns} Q_{ns}\left(K_0^{-n-1,s}\frac{\partial G_s}{\partial h} + h\chi^3 G_s \frac{dK_0^{-n-1,s}}{d\chi}\right) \\
\frac{\partial U}{\partial k} &= -\frac{\mu}{a}\sum_{s,n}(2-\delta_{0s})\left(\frac{R}{a}\right)^n J_n V_{ns} Q_{ns}\left(K_0^{-n-1,s}\frac{\partial G_s}{\partial k} + k\chi^3 \frac{dK_0^{-n-1,s}}{d\chi}\right) \\
\frac{\partial U}{\partial \alpha} &= -\frac{\mu}{a}\sum_{s,n}(2-\delta_{0s})\left(\frac{R}{a}\right)^n J_n V_{ns} K_0^{-n-1,s} Q_{ns}\frac{\partial G_s}{\partial \alpha} \\
\frac{\partial U}{\partial \beta} &= -\frac{\mu}{a}\sum_{s,n}(2-\delta_{0s})\left(\frac{R}{a}\right)^n J_n V_{ns} K_0^{-n-1,s} Q_{ns}\frac{\partial G_s}{\partial \beta} \\
\frac{\partial U}{\partial \gamma} &= -\frac{\mu}{a}\sum_{s,n}(2-\delta_{0s})\left(\frac{R}{a}\right)^n J_n V_{ns} K_0^{-n-1,s}\frac{dQ_{ns}}{d\gamma} G_s
\end{aligned}
$$

Missing G_s term

Recursion for the derivatives of the Hansen coefficient kernels (Danielsen, Sect. 3.1, eq. 7)

$$\frac{dK_0^{-n-1,s}}{d\chi} = \begin{cases} 0 & \text{for } n = s \\ \dfrac{(1+2s)\chi^{2s}}{2^s} & \text{for } n = s+1 \\ \dfrac{(n-1)\chi^2}{(n+s-1)(n-s+1)}\left[(2n-3)\dfrac{dK_0^{-n,s}}{d\chi} - (n-2)\dfrac{dK_0^{-n+1,s}}{d\chi}\right] & \\ \quad + \dfrac{2}{\chi}K_0^{-n-1,s} & \text{for } n > s+1 \end{cases}$$

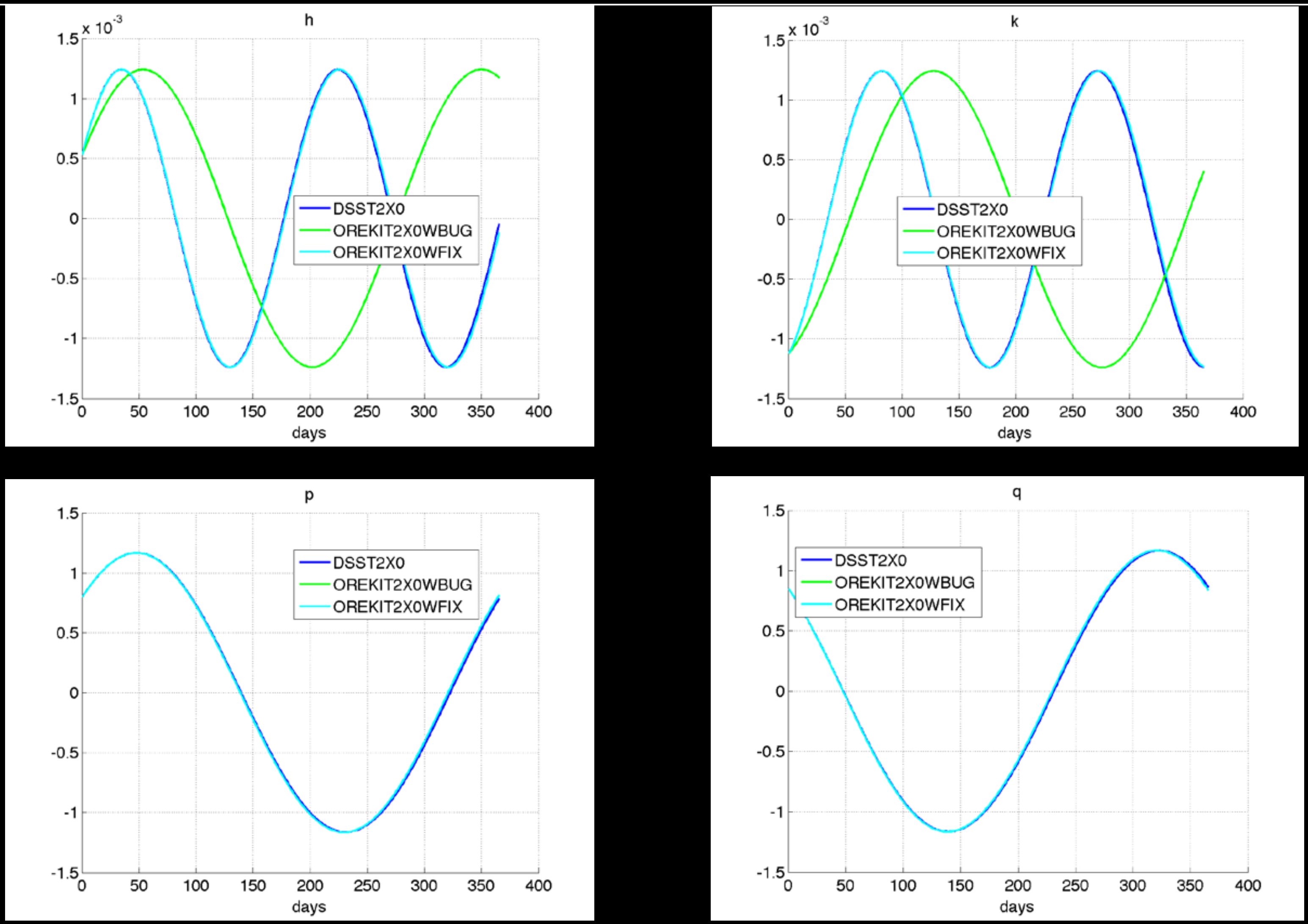
<u>Sign error</u>

- This recursion is obtained by differentiating the recursion for the Hansen kernels
- This algorithm is implemented in Orekit module **HansenCoefficients.java**
- Error in recursion verified by java debug

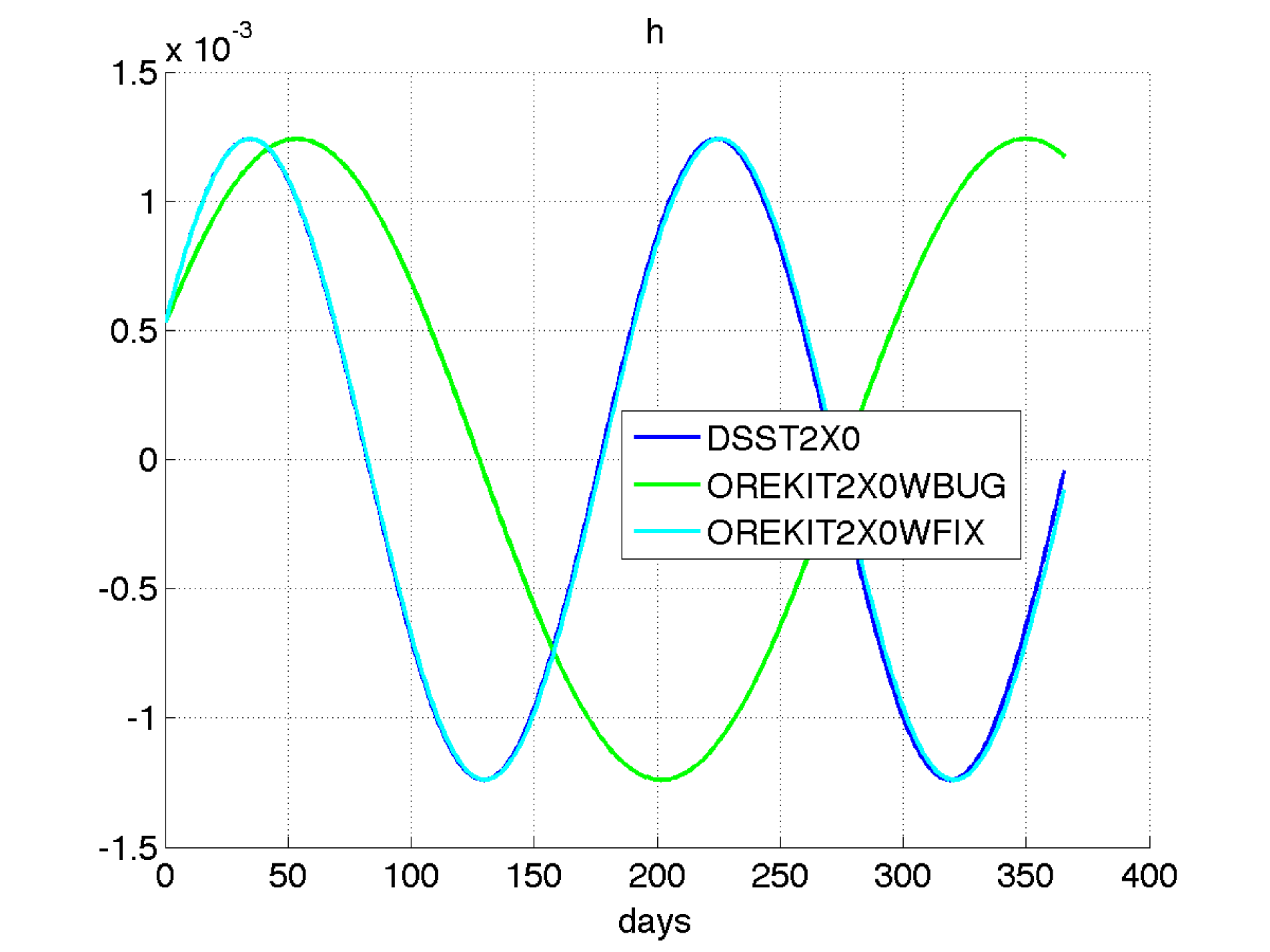
Modification to HansenCoefficients.java

```
private double computeHKDJ0NNegative(final int n, final int s) throws OrekitException {
    double dkdxns = 0.;
    final MNSKey key = new MNSKey(0, -(n + 1), s);

    if (n == FastMath.abs(s)) {
        HANSEN_KERNEL_DERIVATIVES.put(key, 0.);

    } else if (n == FastMath.abs(s) + 1) {
        dkdxns = (1. + 2. * s) * FastMath.pow(chi, 2 * s) / FastMath.pow(2, s);

    } else {

        final MNSKey keymN = new MNSKey(0, -n, s);
        double dkmN = 0.;
        if (HANSEN_KERNEL_DERIVATIVES.containsKey(keymN)) {
            dkmN = HANSEN_KERNEL_DERIVATIVES.get(keymN);
        } else {
            dkmN = computeHKDJ0NNegative(n - 1, s);
        }

        final MNSKey keymNp1 = new MNSKey(0, -n + 1, s);
        double dkmNp1 = 0.;
        if (HANSEN_KERNEL_DERIVATIVES.containsKey(keymNp1)) {
            dkmNp1 = HANSEN_KERNEL_DERIVATIVES.get(keymNp1);
        } else {
            dkmNp1 = computeHKDJ0NNegative(n - 2, s);
        }

        final double kns = getHansenKernelValue(0, -(n + 1), s);

        dkdxns = (n - 1) * chi2 * ((2. * n - 3.) * dkmN - (n - 2.) * dkmNp1) / ((n + s - 1.) * (n - s - 1.));
        dkdxns += 2. * kns / chi;
    }

    HANSEN_KERNEL_DERIVATIVES.put(key, dkdxns);
    return dkdxns;
}
```

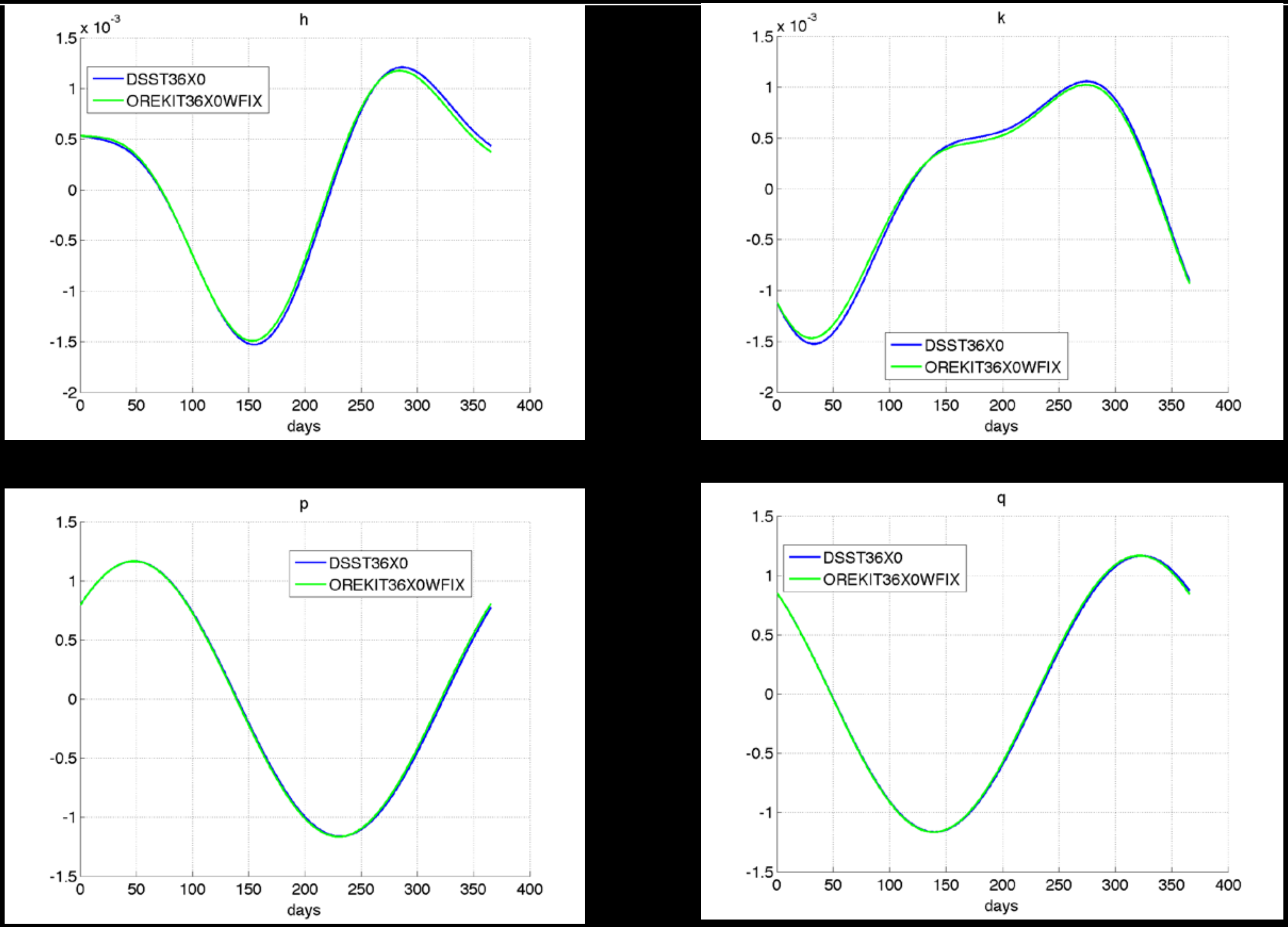
Test 1: LEO, 2 x 0 Equinoctial element histories with fix to Orekit module HansenCoefficients.java



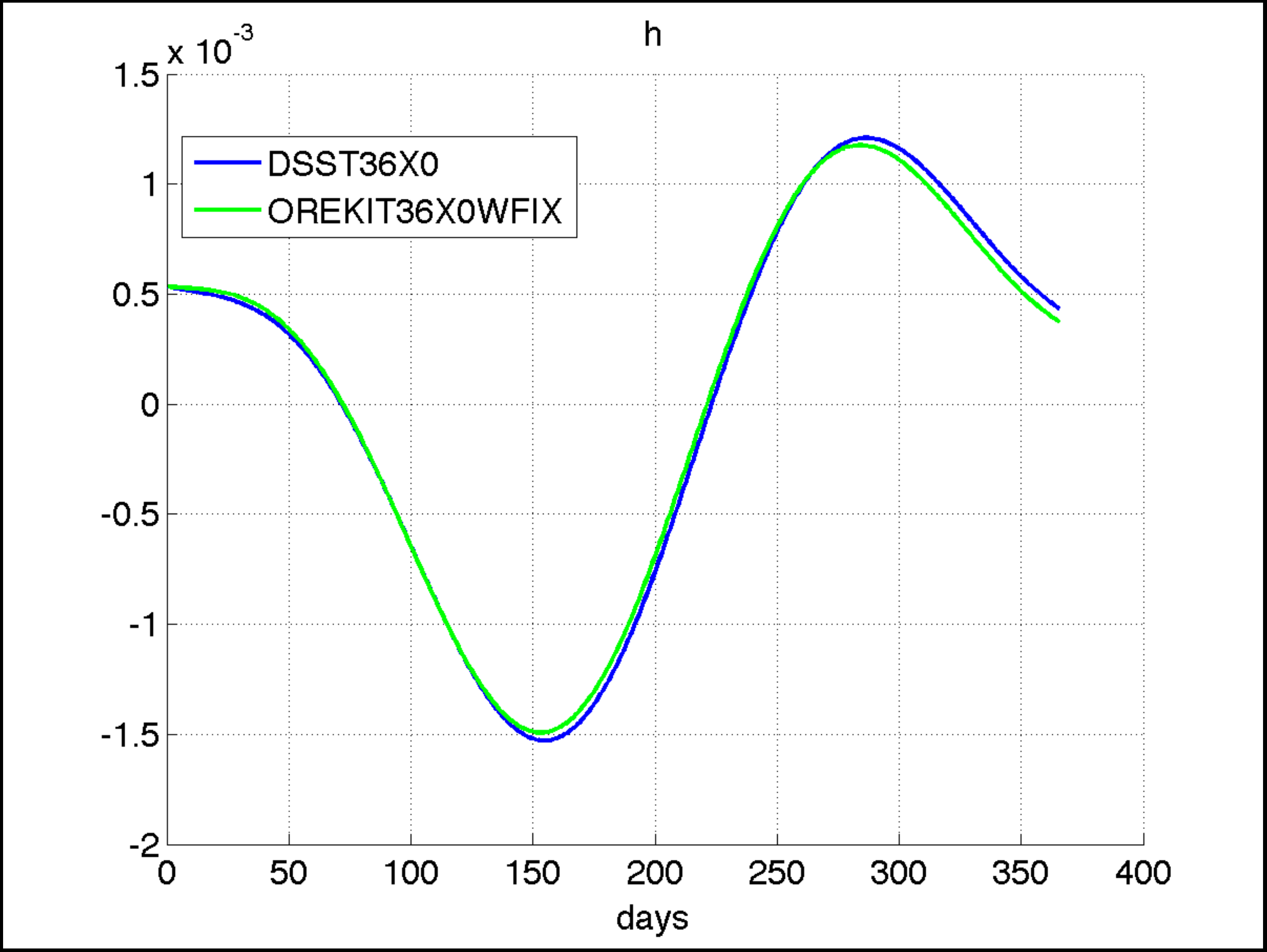
Drilling in on the equinoctial element h time history



Test 1: LEO, 36 x 0 Equinoctial element histories with fix to Orekit module HansenCoefficients.java



Drilling in on the equinoctial element h time history for the 36 x 0 case



Discussion of the Testing Process

- The capability to generate reference trajectories should be shared among the testing players (University of Cambridge, University at Buffalo, CS Systemes, MIT)
- The testing process must include review of the Danielsen document
- Explicit formulas for DSST functionality generated by symbolic algebra systems (Zeis, Kaniecki) can be important.
- It is desirable to create a symbolic algebra version of the Danielsen document
- Matlab can be very helpful
- Testers must be able to make their way through both the Orekit DSST java code and the F77 DSST code
- Testers must understand both the Eclipse java debugger and the Fortran 77 debugger

On-going work

- Debugging the J2-only case resulted in the identification of an error in a Hansen coefficient recursion; the software error propagated from an error in the NPG document
- Need to extend testing to the general zonal harmonic model
- Testing of the lunar-solar point mass model
- Testing of the tesseral resonance model
- Extend the Orekit DSST to include the short-periodic motion and the state transition matrix
- Improve documentation
- Improve the coordination between the Orekit DSST and F77 DSST in time and coordinate systems, physical models

Summary (OREKIT)

- OREKIT is a modern, open-source, extensible, and community-driven astrodynamics library, enabling:
  - Open collaboration
  - Algorithm development
  - Migration of legacy software
  - Standardisation and transparency in SSA

Summary (DSST)

- DSST is a computational efficient theory well suited to SSA
- Previously closed source, now updated and ported to OREKIT
- OREKIT implementation undergoing thorough audit and debugged
- Bugs detected early, and are being worked through to ensure accuracy and stability

Acknowledgements

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Backup slides